Supplementary Back paper Exam
BMath-I Algebra-I
2017-18

Total marks: 100

Time: 3hours

Answer all questions.

(1) (a) Define the action of a group G on a set X. Describe conjugation action of a group G on itself.

(b) Describe the orbit and stabilizer of the matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

under the conjugation action of the group $GL_2(\mathbb{R})$, (invertible 2×2 matrices with entries in $\mathbb{R}$) on itself.

(c) State and prove Cayley's theorem. [4+8+8]

(2) (a) Show that two elements of S_n are conjugate if and only if they have the same cycle type.

(b) Let σ be an m-cycle in S_n, $m \leq n$. Compute the order of the centralizer $C_{S_n}(\sigma)$. Describe the elements of $C_{S_n}(\sigma)$. [10+10]

(3) (a) State and prove *Class equation*.

(b) Determine the possible class equations of a nonabelian group of order 21. Justify your answer. [10+10]

(4) (a) State Sylow's theorems.

(b) Prove that every group of order 15 is cyclic.

(c) Show that a group of order 132 is not simple. [6+6+8]

(5) Find all Sylow subgroups of

(a) A_5 and

(b) S_5. [10+10]

************** *End* **************